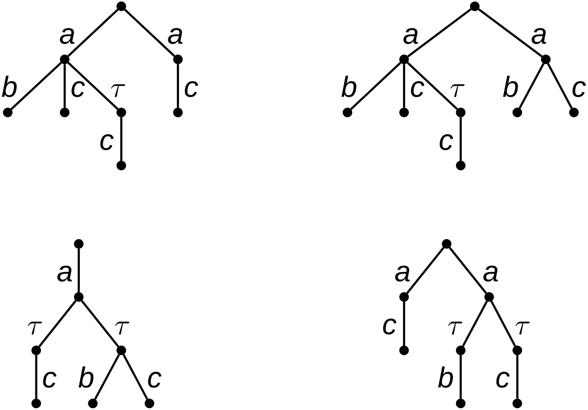
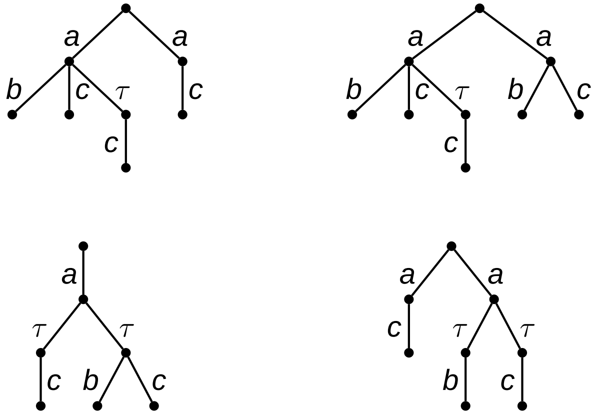
Good old days: Observing trees in JCMB



For fun: add probabilities

Good old days: Observing trees in JCMB



For fun: add probabilities

Some remarks on testing probabilistic processes

Matthew Hennessy

and Yuxin Deng, Rob van Glabbeek, Carroll Morgan, Chenyi Zhang
Nicta

gdp festschrift

Outline

Testing theory

Probabilistic CSP

The power of testing

Simulations

Equational theories

Outline

Testing theory

Probabilistic CSP

The power of testing

Simulations

Equational theories

Outline

Testing theory

Probabilistic CSP

The power of testing

Simulations

Equational theories

Outline

Testing theory

Probabilistic CSP

The power of testing

Simulations

Equational theories

Outline

Outline

Testing scenario

- a set of processes $\mathcal{P}roc$
- a set of tests $\mathcal{T}$
- a set of outcomes $\mathcal{O}$
- $\mathcal{A}pply : \mathcal{T} \times \mathcal{P}roc \rightarrow \mathcal{P}^+_{fin}(\mathcal{O})$ – the non-empty finite set of possible results of applying a test to a process

Comparing sets of outcomes:

- $O_1 \sqsubseteq_{\mathrm{Ho}} O_2$ if for every $o_1 \in O_1$ there exists some $o_2 \in O_2$ such that $o_1 \leq o_2$
- $O_1 \sqsubseteq_{\mathrm{Sm}} O_2$ if for every $o_2 \in O_2$ there exists some $o_1 \in O_1$ such that $o_1 \leq o_2$

$o_1 \leq o_2$: means o_2 is as least as good as o_1

Testing scenario

- a set of processes $\mathcal{P}roc$
- a set of tests $\mathcal{T}$
- a set of outcomes $\mathcal{O}$
- $\mathcal{A}pply : \mathcal{T} \times \mathcal{P}roc \rightarrow \mathcal{P}^+_{fin}(\mathcal{O})$ – the non-empty finite set of possible results of applying a test to a process

Comparing sets of outcomes:

- $O_1 \sqsubseteq_{\mathrm{Ho}} O_2$ if for every $o_1 \in O_1$ there exists some $o_2 \in O_2$ such that $o_1 \leq o_2$
- $O_1 \sqsubseteq_{\mathrm{Sm}} O_2$ if for every $o_2 \in O_2$ there exists some $o_1 \in O_1$ such that $o_1 \leq o_2$

$o_1 \leq o_2$: means o_2 is as least as good as o_1

Testing preorders

- $P \sqsubseteq_{\text{may}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\text{Ho}} \mathcal{A}pply(T, Q)$ for every test T
- $P \sqsubseteq_{\text{must}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\text{Sm}} \mathcal{A}pply(T, Q)$ for every test T

Standard testing:

Use as outcomes $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$

Intuition: with $0 \leq p \leq q \leq 1$, passing a test with probability q better thn passing with probability p

Theorem

For $O_1, O_2 \in \mathcal{P}^+_{fin}(\mathcal{O}_{prob})$ we have

- $O_1 \sqsubseteq_{\text{Ho}} O_2$ *if and only if* $max(O_1) \leq max(O_2)$
- $O_1 \sqsubseteq_{\text{Sm}} O_2$ *if and only if* $min(O_1) \leq min(O_2)$

Testing preorders

- $P \sqsubseteq_{\mathsf{may}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\mathrm{Ho}} \mathcal{A}pply(T, Q)$ for every test T
- $P \sqsubseteq_{\mathsf{must}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\mathrm{Sm}} \mathcal{A}pply(T, Q)$ for every test T

Standard testing:

Use as outcomes $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$

Intuition: with $0 \leq p \leq q \leq 1$, passing a test with probability q better thn passing with probability p

Theorem

For $O_1, O_2 \in \mathcal{P}^{+}_{fin}(\mathcal{O}_{prob})$ we have

- *$O_1 \sqsubseteq_{\mathrm{Ho}} O_2$ if and only if $max(O_1) \leq max(O_2)$*
- *$O_1 \sqsubseteq_{\mathrm{Sm}} O_2$ if and only if $min(O_1) \leq min(O_2)$*

Testing preorders

- $P \sqsubseteq_{may} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{Ho} \mathcal{A}pply(T, Q)$ for every test T
- $P \sqsubseteq_{must} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{Sm} \mathcal{A}pply(T, Q)$ for every test T

Standard testing:

Use as outcomes $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$
Intuition: with $0 \leq p \leq q \leq 1$, passing a test with probability q better thn passing with probability p

Theorem

For $O_1, O_2 \in \mathcal{P}^+_{fin}(\mathcal{O}_{prob})$ we have

- $O_1 \sqsubseteq_{Ho} O_2$ *if and only if* $max(O_1) \leq max(O_2)$
- $O_1 \sqsubseteq_{Sm} O_2$ *if and only if* $min(O_1) \leq min(O_2)$

Testing preorders

- $P \sqsubseteq_{\text{may}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\mathrm{Ho}} \mathcal{A}pply(T, Q)$ for every test T
- $P \sqsubseteq_{\text{must}} Q$ if $\mathcal{A}pply(T, P) \sqsubseteq_{\mathrm{Sm}} \mathcal{A}pply(T, Q)$ for every test T

Standard testing:

Use as outcomes $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$

Intuition: with $0 \leq p \leq q \leq 1$, passing a test with probability q better thn passing with probability p

Theorem

For $O_1, O_2 \in \mathcal{P}^+_{fin}(\mathcal{O}_{prob})$ *we have*

- $O_1 \sqsubseteq_{\mathrm{Ho}} O_2$ *if and only if* $max(O_1) \leq max(O_2)$
- $O_1 \sqsubseteq_{\mathrm{Sm}} O_2$ *if and only if* $min(O_1) \leq min(O_2)$

Outline

Finite probabilistic CSP: pCSP

- $\mathbf{0}$ Do nothing
- $a.P$ Perform a then act as P
- $P \mid_A Q$ Run P and Q in parallel . . .
- $P \Box Q$ External nondeterministic choice between P and Q - environment chooses
- $P \sqcap Q$ Internal nondeterministic choice between P and Q
- $P\ {}_p\oplus\ Q$ Probabilistic choice: act like P with probability p, as Q with probability $(1-p)$

CSP: the sublanguage without probabilistic choice

Finite probabilistic CSP: pCSP

- $\mathbf{0}$ Do nothing
- $a.P$ Perform a then act as P
- $P \mid_A Q$ Run P and Q in parallel …
- $P \Box Q$ External nondeterministic choice between P and Q - environment chooses
- $P \sqcap Q$ Internal nondeterministic choice between P and Q
- $P\,{}_p\!\oplus Q$ Probabilistic choice: act like P with probability p, as Q with probability $(1 - p)$

CSP: the sublanguage without probabilistic choice

Finite probabilistic CSP: pCSP

- $\mathbf{0}$ Do nothing
- $a.P$ Perform a then act as P
- $P \mid_A Q$ Run P and Q in parallel ...
- $P \,\square\, Q$ External nondeterministic choice between P and Q - environment chooses
- $P \sqcap Q$ Internal nondeterministic choice between P and Q
- $P\ {}_p{\oplus}\ Q$ Probabilistic choice: act like P with probability p, as Q with probability $(1-p)$

CSP: the sublanguage without probabilistic choice

Finite probabilistic CSP: pCSP

- $\mathbf{0}$ Do nothing
- $a.P$ Perform a then act as P
- $P\ |_A\ Q$ Run P and Q in parallel ...
- $P \square Q$ External nondeterministic choice between P and Q - environment chooses
- $P \sqcap Q$ Internal nondeterministic choice between P and Q
- $P\ {}_p\!\oplus Q$ Probabilistic choice: act like P with probability p, as Q with probability $(1 - p)$

CSP: the sublanguage without probabilistic choice

Operational semantics

LTS:

a triple $\langle S, \mathsf{Act}_\tau, \rightarrow\rangle$, with

- S a set of states
- Act_τ a set of actions Act, with extra τ
- $\rightarrow \;\subseteq\; S \times \mathsf{Act}_\tau \times S$ - the effect of performing actions.

pLTS, probabilistic LTS:

a triple $\langle S, \mathsf{Act}_\tau, \rightarrow\rangle$, with

- S a set of states
- Act_τ a set of actions Act, with extra τ
- $\rightarrow \;\subseteq\; S \times \mathsf{Act}_\tau \times \mathcal{D}(S)$ - the effect of performing actions

(finite) Distributions $\mathcal{D}(S)$:

Mappings $\Delta : S \rightarrow [0, 1]$ with

- $\sum_{s \in S} \Delta(s) = 1$

Operational semantics

pLTS, probabilistic LTS:

a triple $\langle S, \mathsf{Act}_\tau, \rightarrow \rangle$, with

- S a set of states
- Act_τ a set of actions Act, with extra τ
- $\rightarrow \ \subseteq \ S \times \mathsf{Act}_\tau \times \mathcal{D}(S)$ - the effect of performing actions

(finite) Distributions $\mathcal{D}(S)$:

Mappings $\Delta : S \rightarrow [0,1]$ with

- $\sum_{s \in S} \Delta(s) = 1$
- $\lceil \Delta \rceil := \{ s \in S \mid \Delta(s) > 0 \}$ is finite the support of Δ

Operational semantics

pLTS, probabilistic LTS:

a triple $\langle S, \mathrm{Act}_\tau, \rightarrow\rangle$, with

- S a set of states
- Act_τ a set of actions Act, with extra τ
- $\rightarrow \;\subseteq\; S \times \mathrm{Act}_\tau \times \mathcal{D}(S)$ - the effect of performing actions

(finite) Distributions $\mathcal{D}(S)$:

Mappings $\Delta : S \rightarrow [0,1]$ with

- $\sum_{s \in S} \Delta(s) = 1$
- $\lceil \Delta \rceil := \{\, s \in S \mid \Delta(s) > 0 \,\}$ is finite   the support of Δ

Operational semantics

CSP as an LTS:

- States S are all terms from CSP
- Actions $s \xrightarrow{\alpha} s'$ defined inductively

pCSP as a pLTS:

- States S are subset S_p of terms from pCSP
- Actions $s \xrightarrow{\alpha} s'$ defined inductively
- Terms P in pCSP interpreted as distributions $[\![P]\!]$ over S

pCSP as a pLTS

States S_p:

- $\mathbf{0} \in S_p$
- $a.P \in S_p$
- $P \sqcap Q \in S_p$
- $s_1, s_2 \in S_p$ implies $s_1 \Box s_2 \in S_p$
- $s_1, s_2 \in S_p$ implies $s_1 |_A s_2 \in S_p$.

Distributions $[\![P]\!]$:

- $[\![s]\!] = \overline{s}$ one point distribution $s \rightarrow 1$
- $[\![P\ {}_p\!\oplus Q]\!] = p \cdot [\![P]\!] + (1-p) \cdot [\![Q]\!]$
- $[\![P \Box Q]\!] = [\![P]\!] \ \Box \ [\![Q]\!]$
- $[\![P |_A Q]\!] = [\![P]\!] \ |_A \ [\![Q]\!]$

pCSP as a probabilistic LTS

Defining arrows: $S_p \xrightarrow{\alpha} \mathcal{D}(S_p)$

(ACTION)
$$a.P \xrightarrow{a} [\![P]\!]$$

(INT.L)
$$P \sqcap Q \xrightarrow{\tau} [\![P]\!]$$

(EXT.I.L)
$$\frac{s_1 \xrightarrow{\tau} \Delta}{s_1 \,\Box\, s_2 \xrightarrow{\tau} \Delta \,\Box\, \overline{s_2}}$$

(EXT.L)
$$\frac{s_1 \xrightarrow{a} \Delta}{s_1 \,\Box\, s_2 \xrightarrow{a} \Delta}$$

(PAR.L)
$$\frac{s_1 \xrightarrow{\alpha} \Delta}{s_1 \,|_A\, s_2 \xrightarrow{\alpha} \Delta \,|_A\, \overline{s_2}} \quad \alpha \notin A$$

(PAR.I)
$$\frac{s_1 \xrightarrow{a} \Delta_1, \; s_2 \xrightarrow{a} \Delta_2}{s_1 \,|_A\, s_2 \xrightarrow{\tau} \Delta_1 \,|_A\, \Delta_2} \quad a \in A$$

and symmetric rules

Very similar to standard rules

pCSP as a probabilistic LTS

Defining arrows: $S_p \xrightarrow{\alpha} \mathcal{D}(S_p)$

(ACTION)
$$a.P \xrightarrow{a} [\![P]\!]$$

(INT.L)
$$P \sqcap Q \xrightarrow{\tau} [\![P]\!]$$

(EXT.I.L)
$$\frac{s_1 \xrightarrow{\tau} \Delta}{s_1 \,\Box\, s_2 \xrightarrow{\tau} \Delta \,\Box\, \overline{s_2}}$$

(EXT.L)
$$\frac{s_1 \xrightarrow{a} \Delta}{s_1 \,\Box\, s_2 \xrightarrow{a} \Delta}$$

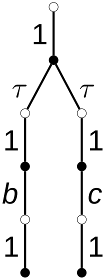
(PAR.L)
$$\frac{s_1 \xrightarrow{\alpha} \Delta}{s_1 \,|_A\, s_2 \xrightarrow{\alpha} \Delta \,|_A\, \overline{s_2}} \quad \alpha \notin A$$

(PAR.I)
$$\frac{s_1 \xrightarrow{a} \Delta_1, \; s_2 \xrightarrow{a} \Delta_2}{s_1 \,|_A\, s_2 \xrightarrow{\tau} \Delta_1 \,|_A\, \Delta_2} \quad a \in A$$

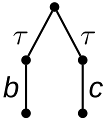
and symmetric rules

Very similar to standard rules

Example pLTSs



abbrev. to

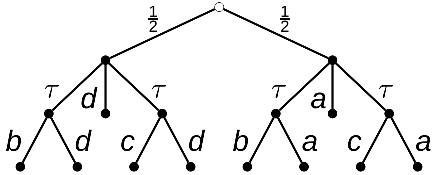


$$b.\mathbf{0} \sqcap c.\mathbf{0}$$

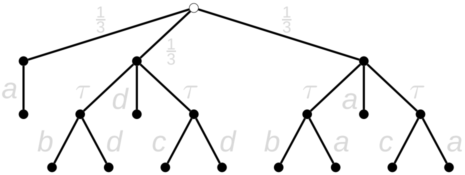
- ● states
- ○ distributions

Example pLTSs

$(b \sqcap c) \,\Box\, (d \;{}_{\frac{1}{2}}\!\oplus\, a)$

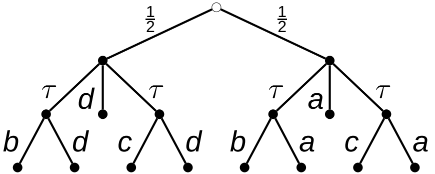


$a \;{}_{\frac{1}{3}}\!\oplus\, ((b \sqcap c) \,\Box\, (d \;{}_{\frac{1}{2}}\!\oplus\, a))$

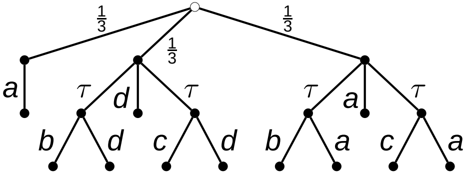


Example pLTSs

$(b \sqcap c) \,\square\, (d \;{}_{\frac{1}{2}}\!\oplus\; a)$



$a \;{}_{\frac{1}{3}}\!\oplus\; ((b \sqcap c) \,\square\, (d \;{}_{\frac{1}{2}}\!\oplus\; a))$



Testing pCSP processes

Tests:

Any process which may contain new *report success* action ω

$a.\omega \;{}_{\frac{1}{4}}\!\oplus (b \,\Box\, c.\omega)$:

- 25% of time requests an a action
- 75% requests a c action
- 75% requires that b is not possible in a must test

Applying test T to process P:

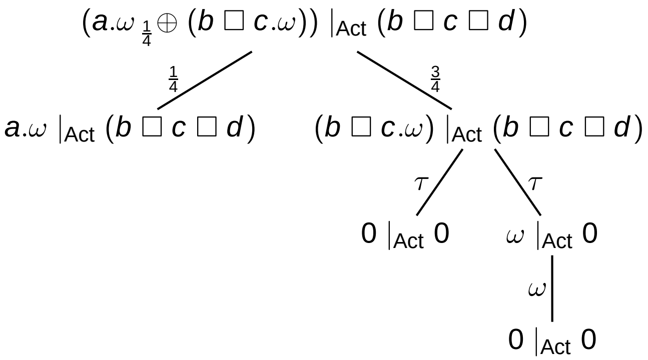
- Run the combined process $T \;|_{\mathrm{Act}}\; P$
- Calculate chances of success

Note: $T \;|_{\mathrm{Act}}\; P$ can only perform τ or *report success*

Running a test

Test: $T = a.\omega \;{}_{\frac{1}{4}}\!\oplus (b \,\Box\, c.\omega)$

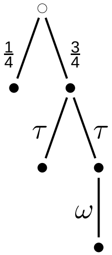
Process $P = b \,\Box\, c \,\Box\, d$



Calculating chances of success

Test: $T = a.\omega \;{}_{\frac{1}{4}}\!\oplus (b \,\Box\, c.\omega)$

Process $P = b \,\Box\, c \,\Box\, d$



Calculate success probabilities:

$$\mathcal{A}pply(T, P) = \frac{1}{4} \cdot \{0\} + \frac{3}{4} \cdot \{0, 1\} = \{0, \frac{3}{4}\}$$

Outline

Testing theory

Probabilistic CSP

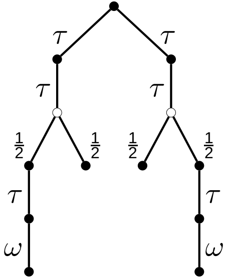
The power of testing

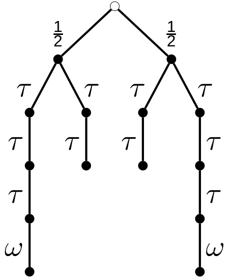
Simulations

Equational theories

Example: probabilistic choice and prefixing

$R_1 = a.(b \,{}_{\frac{1}{2}}\!\oplus c) \qquad R_2 = a.b \,{}_{\frac{1}{2}}\!\oplus a.c \qquad T = a.b.\omega \sqcap a.c.\omega$



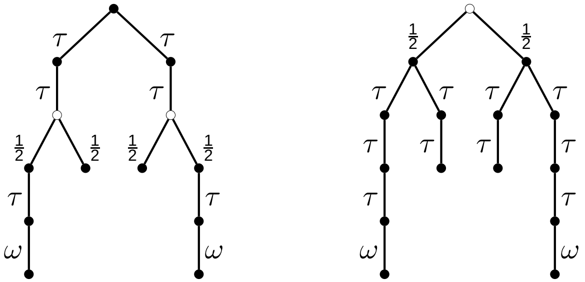


$Apply(T, R_1) = \{\frac{1}{2}\}$ $\qquad$ $Apply(T, R_2) = \{0, \frac{1}{2}, 1\} = \frac{1}{2} \cdot \{0,1\} + \frac{1}{2} \cdot \{0,1\}$

So $R_2 \not\sqsubseteq_{\text{pmay}} R_1$ and $R_1 \not\sqsubseteq_{\text{pmust}} R_2$

Example: probabilistic choice and prefixing

$R_1 = a.(b \,_{\frac{1}{2}}\!\oplus c) \qquad R_2 = a.b \,_{\frac{1}{2}}\!\oplus a.c \qquad T = a.b.\omega \sqcap a.c.\omega$



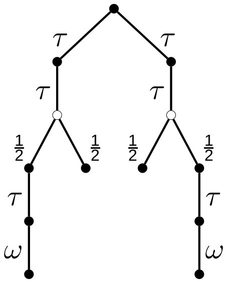
$\mathcal{A}pply(T, R_1) = \{\frac{1}{2}\}$

$\mathcal{A}pply(T, R_2) = \{0, \frac{1}{2}, 1\} = \frac{1}{2} \cdot \{0,1\} + \frac{1}{2} \cdot \{0,1\}$

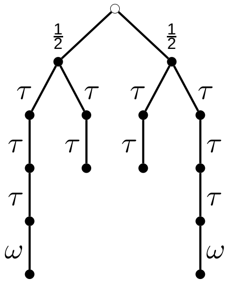
So $R_2 \not\sqsubseteq_{\text{pmay}} R_1$ and $R_1 \not\sqsubseteq_{\text{pmust}} R_2$

Example: probabilistic choice and prefixing

$R_1 = a.(b \,{}_{\frac{1}{2}}\!\oplus c) \qquad R_2 = a.b \,{}_{\frac{1}{2}}\!\oplus a.c \qquad T = a.b.\omega \sqcap a.c.\omega$



$\mathcal{A}pply(T, R_1) = \{\frac{1}{2}\}$

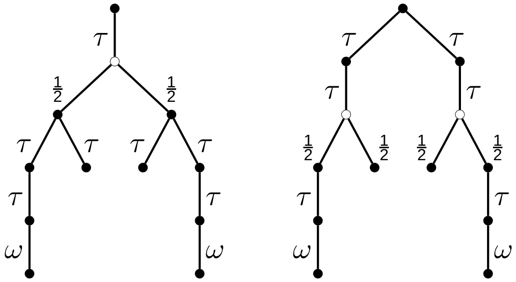


$\mathcal{A}pply(T, R_2) = \{0, \frac{1}{2}, 1\} = \frac{1}{2} \cdot \{0,1\} + \frac{1}{2} \cdot \{0,1\}$

So $R_2 \not\sqsubseteq_{\text{pmay}} R_1$ and $R_1 \not\sqsubseteq_{\text{pmust}} R_2$

A beloved testing axiom

$R_1 = a.(b \sqcap c) \qquad R_2 = a.b \sqcap a.c \qquad T = a.(b.\omega \;_{\frac{1}{2}}\!\oplus c.\omega)$

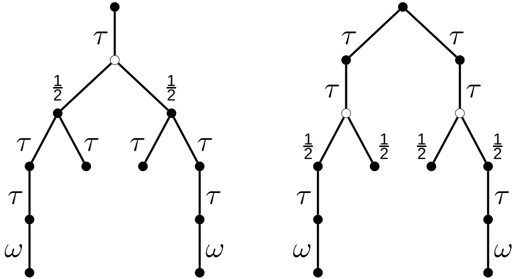


$\mathcal{A}pply(T, R_1) = \{0, \frac{1}{2}, 1\}$ $\qquad$ $\mathcal{A}pply(T, R_2) = \{\frac{1}{2}\}$

So action prefix does not distribute over internal choice

A beloved testing axiom

$R_1 = a.(b \sqcap c) \qquad R_2 = a.b \sqcap a.c \qquad T = a.(b.\omega \;_{\frac{1}{2}}\!\oplus c.\omega)$



$Apply(T, R_1) = \{0, \frac{1}{2}, 1\}$ $\qquad Apply(T, R_2) = \{\frac{1}{2}\}$

So action prefix does not distribute over internal choice

Power of internal choice in tests

- $R_1 = a \,{}_{\frac{1}{2}}\!\oplus (b \,\Box\, c) \qquad R_2 = (a \,\Box\, b) \,{}_{\frac{1}{2}}\!\oplus (a \,\Box\, c)$
- $T = a.(\omega \,{}_{\frac{1}{2}}\!\oplus \mathbf{0}) \sqcap (b.\omega \,{}_{\frac{1}{2}}\!\oplus c.\omega)$

- $\mathcal{A}pply(T, R_1) = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$
- $\mathcal{A}pply(T, R_2) = \{\frac{1}{2}\}$

Therefore $R_1 \not\sqsubseteq_{\text{pmay}} R_2$

Without internal choice in tests, $R_1 \sqsubseteq_{\text{may}} R_2$

Power of internal choice in tests

- $R_1 = a \,{}_{\frac{1}{2}}\!\oplus (b \,\Box\, c) \qquad R_2 = (a \,\Box\, b)\,{}_{\frac{1}{2}}\!\oplus (a \,\Box\, c)$
- $T = a.(\omega \,{}_{\frac{1}{2}}\!\oplus \mathbf{0}) \sqcap (b.\omega \,{}_{\frac{1}{2}}\!\oplus c.\omega)$
- $\mathcal{A}pply(T, R_1) = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\} = \frac{1}{2} \cdot \{0, \frac{1}{2}\} + \frac{1}{2} \cdot \{0, 1\}$
- $\mathcal{A}pply(T, R_2) = \{\frac{1}{2}\} = \frac{1}{2} \cdot \{\frac{1}{2}\} + \frac{1}{2} \cdot \{\frac{1}{2}\}$

Therefore $R_1 \not\sqsubseteq_{\mathrm{pmay}} R_2$

Without internal choice in tests, $R_1 \sqsubseteq_{\mathrm{may}} R_2$

Power of internal choice in tests

- $R_1 = a \, {}_{\frac{1}{2}}\!\oplus (b \, \Box \, c) \qquad R_2 = (a \, \Box \, b) \, {}_{\frac{1}{2}}\!\oplus (a \, \Box \, c)$
- $T = a.(\omega \, {}_{\frac{1}{2}}\!\oplus \mathbf{0}) \sqcap (b.\omega \, {}_{\frac{1}{2}}\!\oplus c.\omega)$

- $\mathcal{A}pply(T, R_1) = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$
- $\mathcal{A}pply(T, R_2) = \{\frac{1}{2}\}$

Therefore $R_1 \not\sqsubseteq_{\text{pmay}} R_2$

Without internal choice in tests, $R_1 \sqsubseteq_{\text{may}} R_2$

Power of internal choice in tests

- $R_1 = a \;_{\frac{1}{2}}\!\oplus (b \,\Box\, c) \qquad R_2 = (a \,\Box\, b) \;_{\frac{1}{2}}\!\oplus (a \,\Box\, c)$
- $T = a.(\omega \;_{\frac{1}{2}}\!\oplus \mathbf{0}) \sqcap (b.\omega \;_{\frac{1}{2}}\!\oplus c.\omega)$
- $\mathcal{A}pply(T, R_1) = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$
- $\mathcal{A}pply(T, R_2) = \{\frac{1}{2}\}$

Therefore $R_1 \not\sqsubseteq_{\text{pmay}} R_2$

Without internal choice in tests, $R_1 \sqsubseteq_{\text{may}} R_2$

Massacre of the axioms

$$a.(P \sqcap Q) \neq a.P \sqcap a.Q$$
$$P \neq P \,\Box\, P$$
$$P \,\Box\, (Q \sqcap R) \neq (P \,\Box\, Q) \sqcap (P \,\Box\, R)$$
$$P \sqcap (Q \,\Box\, R) \neq (P \sqcap Q) \sqcap (P \sqcap R)$$

Hopes dashed:

$$P \;_p\!\oplus (Q \,\Box\, R) \neq (P \;_p\!\oplus Q) \,\Box\, (P \;_p\!\oplus R)$$
$$P \sqcap (Q \;_p\!\oplus R) \neq (P \sqcap Q) \;_p\!\oplus (P \sqcap R)$$
$$P \;_p\!\oplus (Q \,\Box\, R) \neq (P \;_p\!\oplus Q) \,\Box\, (P \;_p\!\oplus R)$$

All is not lost:

$$a.P \,\Box\, a.Q = a.P \sqcap a.Q$$
$$P \,\Box\, (Q \;_p\!\oplus R) = (P \,\Box\, Q) \;_p\!\oplus (P \;_p\!\oplus R)$$
$$\ldots = \ldots$$

Massacre of the axioms

$$a.(P \sqcap Q) \neq a.P \sqcap a.Q$$
$$P \neq P \Box P$$
$$P \Box (Q \sqcap R) \neq (P \Box Q) \sqcap (P \Box R)$$
$$P \sqcap (Q \Box R) \neq (P \sqcap Q) \sqcap (P \sqcap R)$$

Hopes dashed:

$$P\,{}_p\!\oplus (Q \Box R) \neq (P\,{}_p\!\oplus Q) \Box (P\,{}_p\!\oplus R)$$
$$P \sqcap (Q\,{}_p\!\oplus R) \neq (P \sqcap Q)\,{}_p\!\oplus (P \sqcap R)$$
$$P\,{}_p\!\oplus (Q \Box R) \neq (P\,{}_p\!\oplus Q) \Box (P\,{}_p\!\oplus R)$$

All is not lost:

$$a.P \Box a.Q = a.P \sqcap a.Q$$
$$P \Box (Q\,{}_p\!\oplus R) = (P \Box Q)\,{}_p\!\oplus (P\,{}_p\!\oplus R)$$
$$\ldots = \ldots$$

Massacre of the axioms

$$a.(P \sqcap Q) \neq a.P \sqcap a.Q$$
$$P \neq P \,\Box\, P$$
$$P \,\Box\, (Q \sqcap R) \neq (P \,\Box\, Q) \sqcap (P \,\Box\, R)$$
$$P \sqcap (Q \,\Box\, R) \neq (P \sqcap Q) \sqcap (P \sqcap R)$$

Hopes dashed:

$$P \,{}_p\!\oplus (Q \,\Box\, R) \neq (P \,{}_p\!\oplus Q) \,\Box\, (P \,{}_p\!\oplus R)$$
$$P \sqcap (Q \,{}_p\!\oplus R) \neq (P \sqcap Q) \,{}_p\!\oplus (P \sqcap R)$$
$$P \,{}_p\!\oplus (Q \,\Box\, R) \neq (P \,{}_p\!\oplus Q) \,\Box\, (P \,{}_p\!\oplus R)$$

All is not lost:

$$a.P \,\Box\, a.Q = a.P \sqcap a.Q$$
$$P \,\Box\, (Q \,{}_p\!\oplus R) = (P \,\Box\, Q) \,{}_p\!\oplus (P \,{}_p\!\oplus R)$$
$$\ldots = \ldots$$

Outline

Testing theory

Probabilistic CSP

The power of testing

Simulations

Equational theories

Simulations: for may testing

Simulations relate states with distributions

Definition (Segala)

- $\mathcal{R} \subseteq S_p \times \mathcal{D}(S_p)$ is a simulation if
  $s\ \mathcal{R}\ \Delta$ and $s \xrightarrow{\alpha} \Theta$ implies there exists some Δ'
  $\Theta\ \overline{\mathcal{R}}\ \Delta'$ and $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

Requirements:

- Lift $S_p\ \mathcal{R}\ \mathcal{D}(S_p)$ to $\mathcal{D}(S_p)\ \overline{\mathcal{R}}\ \mathcal{D}(S_p)$
- Generalise $s \xrightarrow{\alpha} \Delta'$ to $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

- $\stackrel{\hat{a}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^* \xrightarrow{a} \xrightarrow{\tau}{}^*$
- $\stackrel{\hat{\tau}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^*$

Simulations: for may testing

Simulations relate states with distributions

Definition (Segala)

- $\mathcal{R} \subseteq S_p \times \mathcal{D}(S_p)$ is a simulation if
  $s \; \mathcal{R} \; \Delta$ and $s \xrightarrow{\alpha} \Theta$ implies there exists some Δ'
  $\Theta \; \overline{\mathcal{R}} \; \Delta'$ and $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

Requirements:

- Lift $S_p \;\; \mathcal{R} \;\; \mathcal{D}(S_p)$ to $\mathcal{D}(S_p) \;\; \overline{\mathcal{R}} \;\; \mathcal{D}(S_p)$
- Generalise $s \xrightarrow{\alpha} \Delta'$ to $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

- $\stackrel{\hat{a}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^* \xrightarrow{a} \xrightarrow{\tau}{}^*$
- $\stackrel{\hat{\tau}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^*$

Simulations: for may testing

Simulations relate states with distributions

Definition (Segala)

- $\mathcal{R} \subseteq S_p \times \mathcal{D}(S_p)$ is a simulation if
  $s\ \mathcal{R}\ \Delta$ and $s \xrightarrow{\alpha} \Theta$ implies there exists some Δ'
  $\Theta\ \overline{\mathcal{R}}\ \Delta'$ and $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

Requirements:

- Lift $S_p\ \ \mathcal{R}\ \ \mathcal{D}(S_p)$ to $\mathcal{D}(S_p)\ \ \overline{\mathcal{R}}\ \ \mathcal{D}(S_p)$
- Generalise $s \xrightarrow{\alpha} \Delta'$ to $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$
- $\stackrel{\hat{a}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^{*} \xrightarrow{a} \xrightarrow{\tau}{}^{*}$
- $\stackrel{\hat{\tau}}{\Longrightarrow}$ means $\xrightarrow{\tau}{}^{*}$

Simulations: for may testing

Simulations relate states with distributions

Definition (Segala)

- $\mathcal{R} \subseteq S_p \times \mathcal{D}(S_p)$ is a simulation if
  $s \;\mathcal{R}\; \Delta$ and $s \xrightarrow{\alpha} \Theta$ implies there exists some Δ'
  $\Theta \;\overline{\mathcal{R}}\; \Delta'$ and $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

Requirements:

- Lift $S_p \;\; \mathcal{R} \;\; \mathcal{D}(S_p)$ to $\mathcal{D}(S_p) \;\; \overline{\mathcal{R}} \;\; \mathcal{D}(S_p)$
- Generalise $s \xrightarrow{\alpha} \Delta'$ to $\Delta \stackrel{\hat{\alpha}}{\Longrightarrow} \Delta'$

- $\stackrel{\hat{a}}{\Longrightarrow}$ means $\xrightarrow{\tau}^{*} \xrightarrow{a} \xrightarrow{\tau}^{*}$
- $\stackrel{\hat{\tau}}{\Longrightarrow}$ means $\xrightarrow{\tau}^{*}$

Lifting relation: $S \quad \mathcal{R} \quad \mathcal{D}(S)$

$\Delta_1 \quad \overline{\mathcal{R}} \quad \Delta_2$ if

- source Δ_1 can be decomposed:

$$\Delta_1 = \sum_{i \in I} p_i \cdot \overline{s_i}$$

- each decomposed state s_i can be related:

$$s_i \ \mathcal{R} \ \Phi_i, \quad \text{for some} \quad \Phi_i$$

- related Φ_i yield the target:

$$\Delta_2 = \sum_{i \in I} p_i \cdot \Phi_i$$

Example liftings: actions

$$(a.b \,\Box\, a.c)\ {}_{\frac{1}{2}}\!\oplus\ a.d \quad \xrightarrow{a} \quad b\ {}_{\frac{1}{2}}\!\oplus\ d$$

because $(a.b \,\Box\, a.c) \xrightarrow{a} b \qquad a.d \xrightarrow{a} d$

$$(a.b \,\Box\, a.c)\ {}_{\frac{1}{2}}\!\oplus\ a.d \quad \xrightarrow{a} \quad (b\ {}_{\frac{1}{2}}\!\oplus\ c)\ {}_{\frac{1}{2}}\!\oplus\ d$$

because

- source is $\frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{2} \cdot \overline{a.d}$
- $a.b \,\Box\, a.c \xrightarrow{a} \overline{b} \qquad a.b \,\Box\, a.c \xrightarrow{a} \overline{c} \qquad a.d \xrightarrow{a} \overline{d}$
- target is $\frac{1}{4} \cdot \overline{b} + \frac{1}{4} \cdot \overline{c} + \frac{1}{2} \cdot \overline{d}$

Example liftings: actions

$$(a.b \,\Box\, a.c)\ {}_{\frac{1}{2}}\!\oplus a.d \quad \xrightarrow{a} \quad b\ {}_{\frac{1}{2}}\!\oplus d$$

because $(a.b \,\Box\, a.c) \xrightarrow{a} b \qquad a.d \xrightarrow{a} d$

$$(a.b \,\Box\, a.c)\ {}_{\frac{1}{2}}\!\oplus a.d \quad \xrightarrow{a} \quad (b\ {}_{\frac{1}{2}}\!\oplus c)\ {}_{\frac{1}{2}}\!\oplus d$$

because

- source is $\frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{2} \cdot \overline{a.d}$
- $a.b \,\Box\, a.c \xrightarrow{a} \overline{b} \qquad a.b \,\Box\, a.c \xrightarrow{a} \overline{c} \qquad a.d \xrightarrow{a} \overline{d}$
- target is $\frac{1}{4} \cdot \overline{b} + \frac{1}{4} \cdot \overline{c} + \frac{1}{2} \cdot \overline{d}$

Example liftings: actions

$$(a.b \,\Box\, a.c) \,{}_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad b \,{}_{\frac{1}{2}}\!\oplus\, d$$

because $(a.b \,\Box\, a.c) \xrightarrow{a} b \qquad a.d \xrightarrow{a} d$

$$(a.b \,\Box\, a.c) \,{}_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b \,{}_{\frac{1}{2}}\!\oplus\, c) \,{}_{\frac{1}{2}}\!\oplus\, d$$

because

- source is $\frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{2} \cdot \overline{a.d}$
- $a.b \,\Box\, a.c \xrightarrow{a} \overline{b} \qquad a.b \,\Box\, a.c \xrightarrow{a} \overline{c} \qquad a.d \xrightarrow{a} \overline{d}$
- target is $\frac{1}{4} \cdot \overline{b} + \frac{1}{4} \cdot \overline{c} + \frac{1}{2} \cdot \overline{d}$

Example liftings: actions

$$(a.b \,\Box\, a.c) \;_{\frac{1}{2}}\!\oplus a.d \quad \xrightarrow{a} \quad b \;_{\frac{1}{2}}\!\oplus d$$

because $(a.b \,\Box\, a.c) \xrightarrow{a} b \qquad a.d \xrightarrow{a} d$

$$(a.b \,\Box\, a.c) \;_{\frac{1}{2}}\!\oplus a.d \quad \xrightarrow{a} \quad (b \;_{\frac{1}{2}}\!\oplus c) \;_{\frac{1}{2}}\!\oplus d$$

because

- source is $\frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{4} \cdot \overline{(a.b \,\Box\, a.c)} + \frac{1}{2} \cdot \overline{a.d}$
- $a.b \,\Box\, a.c \xrightarrow{a} \overline{b} \qquad a.b \,\Box\, a.c \xrightarrow{a} \overline{c} \qquad a.d \xrightarrow{a} \overline{d}$
- target is $\frac{1}{4} \cdot \overline{b} + \frac{1}{4} \cdot \overline{c} + \frac{1}{2} \cdot \overline{d}$

Lifting actions: 0 or 1 internal moves: $\xrightarrow{\hat{\tau}}$

$$(a \sqcap b)\ {}_{\frac{1}{2}}\!\oplus (a \sqcap c) \quad \xrightarrow{\hat{\tau}} \quad a\ {}_{\frac{1}{2}}\!\oplus (a \sqcap b\ \ {}_{\frac{1}{2}}\!\oplus\ \ c)$$

because

- source is $\frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap c)} + \frac{1}{4} \cdot \overline{(a \sqcap c)}$
-

$$(a \sqcap b) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap b) \xrightarrow{\hat{\tau}} \overline{(a \sqcap b)}$$
$$(a \sqcap c) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap c) \xrightarrow{\hat{\tau}} \overline{c}$$

- target is $\frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{c}$

Lifting actions: 0 or 1 internal moves: $\xrightarrow{\hat{\tau}}$

$$(a \sqcap b)\,{}_{\frac{1}{2}}\!\oplus (a \sqcap c) \xrightarrow{\hat{\tau}} a\,{}_{\frac{1}{2}}\!\oplus (a \sqcap b\,{}_{\frac{1}{2}}\!\oplus\, c)$$

because

- source is $\frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap c)} + \frac{1}{4} \cdot \overline{(a \sqcap c)}$
-

$$(a \sqcap b) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap b) \xrightarrow{\hat{\tau}} \overline{(a \sqcap b)}$$

$$(a \sqcap c) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap c) \xrightarrow{\hat{\tau}} \overline{c}$$

- target is $\frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{c}$

Lifting actions: 0 or 1 internal moves: $\xrightarrow{\hat{\tau}}$

$$(a \sqcap b)\ {}_{\frac{1}{2}}\!\oplus\ (a \sqcap c) \quad \xrightarrow{\hat{\tau}} \quad a\ {}_{\frac{1}{2}}\!\oplus\ (a \sqcap b\ \ {}_{\frac{1}{2}}\!\oplus\ \ c)$$

because

- source is $\frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap c)} + \frac{1}{4} \cdot \overline{(a \sqcap c)}$
-

$$(a \sqcap b) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap b) \xrightarrow{\hat{\tau}} \overline{(a \sqcap b)}$$
$$(a \sqcap c) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap c) \xrightarrow{\hat{\tau}} \overline{c}$$

- target is $\frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{c}$

Lifting actions: 0 or 1 internal moves: $\xrightarrow{\hat{\tau}}$

$$(a \sqcap b)\ {}_{\frac{1}{2}}\!\oplus (a \sqcap c) \quad \xrightarrow{\hat{\tau}} \quad a\ {}_{\frac{1}{2}}\!\oplus (a \sqcap b\ \ {}_{\frac{1}{2}}\!\oplus\ \ c)$$

because

- source is $\frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{(a \sqcap c)} + \frac{1}{4} \cdot \overline{(a \sqcap c)}$
- $$(a \sqcap b) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap b) \xrightarrow{\hat{\tau}} \overline{(a \sqcap b)}$$
  $$(a \sqcap c) \xrightarrow{\hat{\tau}} \overline{a} \qquad (a \sqcap c) \xrightarrow{\hat{\tau}} \overline{c}$$
- target is $\frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{(a \sqcap b)} + \frac{1}{4} \cdot \overline{a} + \frac{1}{4} \cdot \overline{c}$

Results on simulations

$P \sqsubseteq_S Q$ - lifting of simulations to pCSP processes

- $\sqsubseteq_S$ is a precongruence for pCSP
- For pCSP, $P \sqsubseteq_S Q$ implies $P \sqsubseteq_{\text{pmay}} Q$
- For CSP, $P \sqsubseteq_S Q$ iff $P \sqsubseteq_{\text{pmay}} Q$

- $P \sqsubseteq_S Q$ can be equationally characterised, over pCSP
- $P \sqsubseteq_S Q$ can be equationally characterised, over CSP

Conjecture:

- For pCSP, $P \sqsubseteq_S Q$ iff $P \sqsubseteq_{\text{pmay}} Q$

Results on simulations

$P \sqsubseteq_S Q$ - lifting of simulations to pCSP processes

- $\sqsubseteq_S$ is a precongruence for pCSP
- For pCSP, $P \sqsubseteq_S Q$ implies $P \sqsubseteq_{\mathrm{pmay}} Q$
- For CSP, $P \sqsubseteq_S Q$ iff $P \sqsubseteq_{\mathrm{pmay}} Q$

- $P \sqsubseteq_S Q$ can be equationally characterised, over pCSP
- $P \sqsubseteq_S Q$ can be equationally characterised, over CSP

Conjecture:

- For pCSP, $P \sqsubseteq_S Q$ iff $P \sqsubseteq_{\mathrm{pmay}} Q$

Outline

Equations for CSP

tests contain probability choice, processes not

A selection:

$$
\begin{aligned}
a.P \,\Box\, a.Q &= a.P \sqcap a.Q \\
a.P \,\Box\, (Q \sqcap R) &= (a.P \,\Box\, Q) \sqcap (a.P \,\Box\, R) \\
P \,\Box\, Q &= (P_1 \,\Box\, Q) \sqcap (P_2 \,\Box\, Q) \sqcap (P \,\Box\, Q_1) \sqcap (P \,\Box\, Q_2),
\end{aligned}
$$

provided $P = P_1 \sqcap P_2,\; Q = Q_1 \sqcap Q_2$

For completeness add: $P \sqsubseteq P \sqcap Q \qquad P \sqcap Q = P \,\Box\, Q$ standard

Interesting derived equations:

$$
\begin{aligned}
a.P \sqcap a.Q &\sqsubseteq a.(P \sqcap Q) \\
(P \,\Box\, Q) \sqcap (P \,\Box\, R) &\sqsubseteq P \,\Box\, (Q \sqcap R) \\
P \sqcap (Q \,\Box\, R) &\sqsubseteq (P \sqcap Q) \,\Box\, (P \sqcap R)
\end{aligned}
$$

Equations for CSP

tests contain probability choice, processes not

A selection:

$$\begin{aligned} a.P \,\Box\, a.Q &= a.P \sqcap a.Q \\ a.P \,\Box\, (Q \sqcap R) &= (a.P \,\Box\, Q) \sqcap (a.P \,\Box\, R) \\ P \,\Box\, Q &= (P_1 \,\Box\, Q) \sqcap (P_2 \,\Box\, Q) \sqcap (P \,\Box\, Q_1) \sqcap (P \,\Box\, Q_2), \end{aligned}$$

provided $P = P_1 \sqcap P_2,\; Q = Q_1 \sqcap Q_2$

For completeness add: $P \sqsubseteq P \sqcap Q \qquad P \sqcap Q = P \,\Box\, Q$ standard

Interesting derived equations:

$$\begin{aligned} a.P \sqcap a.Q &\sqsubseteq a.(P \sqcap Q) \\ (P \,\Box\, Q) \sqcap (P \,\Box\, R) &\sqsubseteq P \,\Box\, (Q \sqcap R) \\ P \sqcap (Q \,\Box\, R) &\sqsubseteq (P \sqcap Q) \,\Box\, (P \sqcap R) \end{aligned}$$

Equations for CSP

tests contain probability choice, processes not

A selection:

$$
\begin{aligned}
a.P \,\Box\, a.Q &= a.P \sqcap a.Q \\
a.P \,\Box\, (Q \sqcap R) &= (a.P \,\Box\, Q) \sqcap (a.P \,\Box\, R) \\
P \,\Box\, Q &= (P_1 \,\Box\, Q) \sqcap (P_2 \,\Box\, Q) \sqcap (P \,\Box\, Q_1) \sqcap (P \,\Box\, Q_2),
\end{aligned}
$$

provided $P = P_1 \sqcap P_2,\ Q = Q_1 \sqcap Q_2$

For completeness add: $P \sqsubseteq P \sqcap Q$ $\quad P \sqcap Q = P \,\Box\, Q$ standard

Interesting derived equations:

$$
\begin{aligned}
a.P \sqcap a.Q &\sqsubseteq a.(P \sqcap Q) \\
(P \,\Box\, Q) \sqcap (P \,\Box\, R) &\sqsubseteq P \,\Box\, (Q \sqcap R) \\
P \sqcap (Q \,\Box\, R) &\sqsubseteq (P \sqcap Q) \,\Box\, (P \sqcap R)
\end{aligned}
$$

Equations for pCSP

processes and tests contain probability choice

Standard:

$$P\ {}_{p}\!\oplus P = P$$
$$P\ {}_{p}\!\oplus Q = Q\ {}_{1-p}\!\oplus P$$
$$(P\ {}_{p}\!\oplus Q)\ {}_{q}\!\oplus R = P\ {}_{p\cdot q}\!\oplus (Q\ {}_{\frac{(1-p)\cdot q}{1-p\cdot q}}\!\oplus R)$$

Add:

$$a.(P\ {}_{p}\!\oplus Q) \sqsubseteq a.P\ {}_{p}\!\oplus a.Q$$
$$P \Box (Q\ {}_{p}\!\oplus R) = (P \Box Q)\ {}_{p}\!\oplus (P \Box R)$$

Interesting derived equations:

$$(P\ {}_{p}\!\oplus Q) \sqcap (P\ {}_{p}\!\oplus R) \sqsubseteq P\ {}_{p}\!\oplus (Q \sqcap R)$$
$$P \sqcap (Q\ {}_{p}\!\oplus R) \sqsubseteq (P \sqcap Q)\ {}_{p}\!\oplus (P \sqcap R)$$
$$a.(P\ {}_{p}\!\oplus Q) \sqsubseteq a.P \Box a.Q$$

Equations for pCSP

processes and tests contain probability choice

Standard:

$$\begin{aligned} P\ {}_p\!\oplus P &= P \\ P\ {}_p\!\oplus Q &= Q\ {}_{1-p}\!\oplus P \\ (P\ {}_p\!\oplus Q)\ {}_q\!\oplus R &= P\ {}_{p \cdot q}\!\oplus (Q\ {}_{\frac{(1-p)\cdot q}{1-p\cdot q}}\!\oplus R) \end{aligned}$$

Add:

$$\begin{aligned} a.(P\ {}_p\!\oplus Q) &\sqsubseteq a.P\ {}_p\!\oplus a.Q \\ P \,\Box\, (Q\ {}_p\!\oplus R) &= (P \,\Box\, Q)\ {}_p\!\oplus (P \,\Box\, R) \end{aligned}$$

Interesting derived equations:

$$\begin{aligned} (P\ {}_p\!\oplus Q) \sqcap (P\ {}_p\!\oplus R) &\sqsubseteq P\ {}_p\!\oplus (Q \sqcap R) \\ P \sqcap (Q\ {}_p\!\oplus R) &\sqsubseteq (P \sqcap Q)\ {}_p\!\oplus (P \sqcap R) \\ a.(P\ {}_p\!\oplus Q) &\sqsubseteq a.P \,\Box\, a.Q \end{aligned}$$

Equations for pCSP

processes and tests contain probability choice

Standard:

$$P \,{}_p\!\oplus P = P$$
$$P \,{}_p\!\oplus Q = Q \,{}_{1-p}\!\oplus P$$
$$(P \,{}_p\!\oplus Q) \,{}_q\!\oplus R = P \,{}_{p \cdot q}\!\oplus \left(Q \,{}_{\frac{(1-p)\cdot q}{1-p\cdot q}}\!\oplus R\right)$$

Add:

$$a.(P \,{}_p\!\oplus Q) \sqsubseteq a.P \,{}_p\!\oplus a.Q$$
$$P \,\Box\, (Q \,{}_p\!\oplus R) = (P \,\Box\, Q) \,{}_p\!\oplus (P \,\Box\, R)$$

Interesting derived equations:

$$(P \,{}_p\!\oplus Q) \sqcap (P \,{}_p\!\oplus R) \sqsubseteq P \,{}_p\!\oplus (Q \sqcap R)$$
$$P \sqcap (Q \,{}_p\!\oplus R) \sqsubseteq (P \sqcap Q) \,{}_p\!\oplus (P \sqcap R)$$
$$a.(P \,{}_p\!\oplus Q) \sqsubseteq a.P \,\Box\, a.Q$$

The end

Lots of work to do:

- Prove relationship between simulations and may testing
- Must testing:
  - relate to *failure simulations*
  - equations ?
- Modal logics ?

Related work

- Segala, Wang Yi, Jonsson, Larsen, Skou, Lowe, Cleaveland, Smolka, . . .
- Google:
  - probabilistic process calculi: 631,000
  - testing probabilistic process calculi: 394,000
  - testing probabilistic processes: 4,800,000
  - probabilistic processes: 7,200,000

The end

Lots of work to do:

- Prove relationship between simulations and may testing
- Must testing:
  - relate to *failure simulations*
  - equations ?
- Modal logics ?

Related work

- Segala, Wang Yi, Jonsson, Larsen, Skou, Lowe, Cleaveland, Smolka, . . .
- Google:
  - probabilistic process calculi: 631,000
  - testing probabilistic process calculi: 394,000
  - testing probabilistic processes: 4,800,000
  - probabilistic processes: 7,200,000